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Electroosmotic and Thermal Analysis of Magnetohydrodynamic Couple-Stress Hybrid Nanofluid Flow in a Porous Medium with Hall and Ion-Slip Effects

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Abstract.

This study presents a comprehensive thermal analysis of electroosmotic magnetohydrodynamic (MHD) couple-stress hybrid nanofluid flow in a porous medium, incorporating Hall and ion-slip effects. The combined influence of electroosmosis and MHD interactions enhances fluid transport, making it highly relevant for advanced microfluidic and energy applications. A mathematical model is developed using the governing equations of fluid motion, heat, and mass transfer, incorporating couple-stress effects to capture the non-Newtonian behavior of the fluid. The electroosmotic force, induced by an external electric field, modulates the velocity and temperature profiles, while the Hall and ion-slip effects further influence the transport characteristics. Analytical and numerical techniques are employed to solve the transformed boundary value problem, providing insights into the impact of key parameters on velocity, temperature, and concentration distributions. The results reveal that electroosmotic effects significantly enhance flow control and thermal efficiency, which is crucial for biomedical, filtration, and energy conversion systems. A parametric study demonstrates the sensitivity of the system to variations in electroosmotic strength, magnetic field, and couple-stress parameters. The findings contribute to the optimization of hybrid nanofluid-based thermal management systems, offering new perspectives for next-generation microfluidic technologies.

Keywords:
Electroosmotic flow, Magnetohydrodynamics, Couple-stress fluid, Hybrid nanofluid

Introduction

The study of magnetohydrodynamic (MHD) flows in electrically conducting fluids has garnered significant attention due to its applications in energy systems, biomedical engineering, and microfluidic technologies. Integrating electroosmotic flow (EOF) with MHD principles enhances control over transport phenomena, particularly in microelectromechanical systems (MEMS), lab-on-a-chip devices, and filtration systems, where the interplay of electroosmosis, magnetic fields, and thermal transport optimizes performance (Sadeghy *et al*., 2005; Bhatti *et al*., 2021).

Electroosmotic flow (EOF), a cornerstone of electrokinetics, arises from the interaction between an external electric field and the electric double layer (EDL) at a solid-liquid interface. When an electrolyte contacts a charged surface, ionization creates a surface charge balanced by counter-ions, forming the EDL and establishing a zeta potential. Applying an electric field mobilizes ions in the EDL's diffuse layer, generating bulk fluid motion via viscous coupling (Kirby, 2010). This mechanism enables precise fluid control in micro- and nanofluidic systems, revolutionizing drug delivery, bioengineering, and electroosmotic pumps (Bruus, 2008; Ghosh & Chakraborty, 2019). Recent advancements combine EOF with MHD to form electro-magnetohydrodynamic (EMHD) systems, which enhance microchannel pumping efficiency and thermal management (Shah *et al*., 2022).

Hybrid nanofluids (HNFs), pioneered by Choi and Eastman (1995), utilize synergistic combinations of nanoparticles (e.g., SWCNT/MWCNT) suspended in base fluids like ethylene glycol to improve thermal conductivity and energy transport. These fluids are pivotal in cooling technologies, energy storage, and biomedical applications due to their enhanced thermal stability and rheological properties (Rashidi *et al*., 2016; Sajid & Ali, 2018). For instance, SWCNT/MWCNT-ethylene glycol HNFs exhibit superior heat transfer in EMHD microchannels, attributed to their high aspect ratios and reduced defect density (Zhao *et al*., 2021).

Couple-stress fluids (CSFs), introduced by Stokes (1966), extend classical fluid dynamics by modeling microstructural effects in non-Newtonian fluids such as blood, polymeric solutions, and lubricants. Their constitutive equations incorporate asymmetric stress tensors and rotational couple stresses, enabling predictions of size-dependent flow behavior (Stokes, 1966). Recent studies integrate CSF models with HNFs in EMHD systems, demonstrating enhanced heat transfer under electroosmotic modulation and porous media interactions (Jangili *et al*., 2020; Shafee *et al*., 2023; Zhang *et al*., 2022).

In high magnetic fields, Hall and ion-slip effects significantly alter ion trajectories, leading to charge separation and drift. These phenomena, critical in plasma physics and astrophysical flows, influence MHD transport and heat transfer in hybrid nanofluids (Agarwal *et al*., 2018). For example, Hall effects disrupt boundary layer dynamics, while ion-slip effects improve thermal characteristics in magnetized HNF flows (Rashidi *et al*., 2016; Tiwari & Das, 2007). When combined with EOF in porous media, these effects introduce complex transport dynamics, particularly in vertical channels where buoyancy-driven convection interacts with electrokinetic forces (Mushahary & Ontela, 2023).

Porous media further modulate fluid dynamics through permeability-dependent Darcy resistance, playing a vital role in filtration, catalysis, and biomedical systems. Recent studies emphasize optimizing porosity and Darcy numbers to enhance heat and mass transfer in HNFs under electroosmotic-MHD interactions (Nield & Bejan, 2017; Bhatti *et al*., 2021). Temperature-dependent electrical conductivity and variable permeability add complexity, necessitating advanced models to predict flow and entropy generation (Adesanaya *et al*., 2022).

Entropy generation minimization (EGM), conceptualized by Bejan (1996), provides a framework to optimize thermal systems by quantifying irreversibilities from heat transfer, fluid friction, and Joule dissipation. In CSF-HNFs, variable electrical conductivity and porous permeability significantly affect entropy production, highlighting the need for parametric optimization in EMHD applications (Gireesha *et al*., 2018; Mushahary & Ontela, 2023).

Research Gap and Objectives

Despite advancements, the interplay of EOF, EMHD, CSF-HNFs, and entropy generation in vertical porous channels remains underexplored. This study addresses the following objectives:

1. Analyze the synergistic effects of EOF and EMHD forces on CSF-HNF velocity and temperature profiles.
2. Assess the impact of temperature-dependent electrical conductivity and porous permeability on heat transfer and entropy generation.
3. Evaluate the influence of zeta potential and Debye-Hückel parameters on flow irreversibility.
4. Quantify SWCNT/MWCNT concentration effects on thermal performance.

2. Mathematical Formulations and Governing Equations

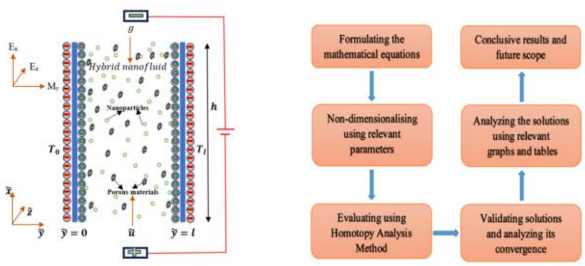

Figure1: The Geometry of the Flow

Continuity Equation

For incompressible flow:

$$\nabla \cdot \mathbf{u} = 0 \quad 1$$

Munson et al. 2021, White, F.M 2021, (standard fluid mechanics).

Momentum Equation

Incorporating couple-stress, electroosmotic, Lorentz, and Darcy forces:

$$0 = -\frac{dp}{dx} + \mu_{\mathrm{nf}}\frac{d^2u}{dy^2} - \eta\frac{d^4u}{dy^4} + \rho_e E_x + \left(J_y B_0\right) - \frac{\mu_{\mathrm{nf}}}{K}u \quad 5$$

Couple-stress term: Stokes (1966); Hall and ion-slip effects, Darcy resistance: Agarwal et al. (2018), Nield and Bejan (2017).

Energy Equation

Including conduction, viscous dissipation, couple-stress dissipation, and Joule heating:

$$0 = k_{\text{nf}}\frac{d^2T}{dy^2} + \mu_{\text{nf}}\left(\frac{du}{dy}\right)^2 + \eta\left(\frac{d^2u}{dy^2}\right)^2 + J\cdot E \quad 2$$

Nanofluid thermal conductivity: Choi and Eastman (1995); Joule heating: Agarwal et al. (2018).

Electric Potential (Poisson-Boltzmann Equation)

Under Debye-Hückel approximation:

$$\frac{d^2\psi}{dy^2} = \kappa^2\psi \quad 3$$

Kirby (2010).

Current Density with Hall and Ion-Slip Effects

Generalized Ohm's law:

$$J_x = \frac{\sigma_{\text{nf}}(E_x+uB_0)}{1+\beta_h^2+\beta_i}, \quad J_y = \frac{-\sigma_{\text{nf}}\beta_h(E_x+uB_0)}{1+\beta_h^2+\beta_i} \quad 4$$

Agarwal et al. (2018).

Nanofluid Effective Properties

- Density: $\rho_{\text{nf}} = (1-\phi)\rho_f + \phi\rho_p$
- Viscosity: $\mu_{\text{nf}} = \mu_f(1-\phi)^{-2.5}$
- Thermal conductivity: Maxwell model
- Electrical conductivity: Maxwell model

  Choi and Eastman (1995).

Non-Dimensionalization

Dimensionless Variables:

$$y^* = \frac{y}{h}, \quad u^* = \frac{u}{U}, \quad \psi^* = \frac{\psi}{\zeta}, \quad T^* = \frac{T-T_0}{\Delta T} \quad 6$$

Dimensionless Parameters:

Parameter	Symbol	Expression
Couple-stress	C	$\frac{\eta}{\mu_{\text{nf}}h^2}$ or $\frac{\eta}{\mu_{\text{nf}}}\cdot h^2$
Hartmann	Ha	$B_0 h\sqrt{\sigma_{\text{nf}}/\mu_{\text{nf}}}$
Hall	β_h	Dimensionless
Ion-slip	β_i	Dimensionless
Darcy	Da	K/h^2
Electroosmotic	EO	$\frac{\epsilon\kappa^2\zeta E_x h^2}{\mu_{\text{nf}}U}$

Table 1: Dimensionless Parameters:

Dimensionless Governing Equations

Using the Dimensionless Variables in equation (6), (2), (3) and (4) becomes (7), (8) and (9) respectively.

Momentum:

$$P + \frac{d^2u^*}{dy^{*2}} - C\frac{d^4u^*}{dy^{*4}} + EO\psi^* - M(\beta_h u^*) - Da^{-1}u^* = 0 \quad 7$$

Energy:

$$\frac{d^2T^*}{dy^{*2}} + Br\left(\frac{du^*}{dy^*}\right)^2 + Br_c\left(\frac{d^2u^*}{dy^{*2}}\right)^2 + SrJ_x^* = 0 \quad 8$$

Parameter	Symbol	Physical Meaning
Couple-stress	C	Ratio of couple-stress to viscous effects
Hartmann	M	Magnetic field strength
Darcy	Da	Porous medium permeability
Brinkman	Br	Viscous dissipation
Couple-stress Brinkman	Br_c	Couple-stress dissipation
Joule heating	Sr	Electrical heating effects

Electric Potential:

$$\frac{d^2\psi^*}{dy^{*2}} = (\kappa h)^2\psi^* \qquad 9$$

Boundary Conditions (Dimensionless):

Velocity: $u^*(0) = u^*(1) = 0$, $\frac{d^2u^*}{dy^{*2}} = 0$ at walls.

Potential: $\psi^*(0) = \psi^*(1) = 1$. 10

Temperature: $T^*(0) = T^*(1) = 0$.

Similarity Solutions

To convert (7), (8) and (9) to ordinary differential equations, we introduce similarity condition, $\eta = y^*$ and redefine dependent variables, then (7), (8) and (9) modifies to (11), (12) and (13) respectively.

Momentum Ordinary Differential Equation:

$$-Cf''''(\eta) + f''(\eta) - (M\beta_h + Da^{-1})f(\eta) + EO\phi(\eta) + P = 0 \qquad 11$$

with boundary conditions: $f(0) = f(1) = 0, \quad f''(0) = f''(1) = 0$.

Energy Ordinary Differential Equation:

$$\theta''(\eta) + Br\left(f'(\eta)\right)^2 + Br_c\left(f''(\eta)\right)^2 + SrJ_x^* = 0 \qquad 12$$

with boundary conditions: $\theta(0) = \theta(1) = 0$.

Electric Potential Ordinary Differential Equation:

$$\phi''(\eta) - (\kappa h)^2\phi(\eta) = 0 \qquad 13$$

with boundary conditions: $\phi(0) = \phi(1) = 1$.

Key Parameters

3. Mathematical Methodology

To solve the transformed boundary value problem for the electroosmotic MHD couple-stress hybrid nanofluid flow, the following method of solutions is employed:

3.1. Analytical Solution for Electric Potential

Governing Equation (Dimensionless):

$$\phi''(\eta) - (\kappa h)^2\phi(\eta) = 0 \qquad 14$$

Boundary Conditions:

$$\phi(0) = \phi(1) = 1.$$

This is a linear second-order ODE. Under the Debye-Hückel approximation, the analytical solution is:

$$\phi(\eta) = \frac{\sinh(\kappa h\eta)}{\sinh(\kappa h)}. \qquad 15$$

This satisfies the boundary conditions and describes the electric potential distribution across the channel.

3.2. Numerical Solution for Momentum Equation

Governing Equation (Dimensionless):

$$-Cf''''(\eta) + f''(\eta) - (M\beta_h + Da^{-1})f(\eta) + EO\phi(\eta) + P = 0 \qquad 16$$

Boundary Conditions:

$$f(0) = f(1) = 0, \quad f''(0) = f''(1) = 0.$$

Method:

- Substitute the analytical solution for $\phi(\eta)$ into Equation 13.

Reduction of Fourth-Order ODE to First-Order System

Define the variables:

- $y_1 = f$
- $y_2 = f'$
- $y_3 = f''$
- $y_4 = f'''$

The system becomes:

$$\begin{cases} y_1' = y_2, \\ y_2' = y_3, \\ y_3' = y_4, \\ y_4' = \frac{1}{C}[y_3 - (M\beta_h + Da^{-1})y_1 + EO\,\phi(\eta) + P]. \end{cases} \quad 17$$

Use the shooting method with the Runge-Kutta-Fehlberg (RKF45) algorithm to iteratively adjust initial guesses for $f'''(0)$ and $f'''(1)$ until all boundary conditions are satisfied.

3.3. Numerical Solution for Energy Equation

Governing Equation (Dimensionless):

$$\theta''(\eta) + Br\big(f'(\eta)\big)^2 + Br_c\big(f''(\eta)\big)^2 + SrJ_x^* = 0 \quad 18$$

Boundary Conditions:

$$\theta(0) = \theta(1) = 0.$$

Method:

Substitute the numerically obtained $f(\eta)$ and $f''(\eta)$ into Equation 18.

Solve the second-order ODE using the finite difference method:

Discretize the domain $0 \le \eta \le 1$ into N nodes. Approximate derivatives using central differences. and formulate a tridiagonal system of equations and solve using the Thomas algorithm.

3.4. Validation and Parametric Study

3.4.1. Convergence Check:

Ensure grid independence by refining the mesh until results stabilize (e.g., $N = 1000$ nodes).

Validate numerical solutions against simplified analytical cases (e.g., vanishing couple-stress $C \to 0$ or no magnetic field $M \to 0$).

3.4.2 Parametric Sensitivity:

Vary key parameters $(EO, M, C, Da, \beta_h, \beta_i)$ to analyze their impact on velocity $f(\eta)$, temperature $\theta(\eta)$, and entropy generation.

3.4.3 Key Numerical Tools

Software: MAPLE, **Algorithms:**

Shooting method for high-order ODEs. Finite difference method for linearized equations.

Tridiagonal matrix solver for discretized systems.

5. Summary of Steps

Analytical Step: Solve $\phi(\eta)$ using hyperbolic functions. Momentum Step: Solve $f(\eta)$ numerically via shooting method. Energy Step: Solve $\theta(\eta)$ using finite differences.

Post-Processing: Analyze flow, thermal profiles, and irreversibility. This hybrid analytical-numerical approach efficiently captures the coupled electroosmotic-MHD effects in porous media, enabling optimization of hybrid nanofluid-based systems.

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Figure 2: *Velocity Plot*

Figure 3: *Temperature Plot*

Figure 2. Velocity Plot it illustrates how the fluid flow is influenced by Electroosmotic Force: Generated by the interaction of an external electric field with the electric double layer (EDL) at the channel walls. This drives bulk fluid motion and the Magnetohydrodynamic (MHD) Effects: The Lorentz force (from the magnetic field B_0) opposes the flow, reducing velocity. i. Couple-Stress Effects: Non-Newtonian behavior introduces microstructural resistance, captured by the couple-stress parameter C. ii. Porous Medium Resistance: Darcy resistance ($\propto Da^{-1}$) slows the flow in permeable media and the Hall and Ion-Slip Effects: Modify current density, altering Lorentz force magnitude and direction, it Optimizes flow control in applications like microfluidic pumps, filtration systems, or lab-on-a-chip devices and Helps balance competing forces (e.g., electroosmotic vs. magnetic damping) to achieve desired flow rates. While figure 3 is the temperature plot reflects energy transfer mechanisms, Viscous Dissipation (Br): Friction between fluid layers generates heat, Couple-Stress Dissipation (Br_c): Energy loss due to microstructural interactions in non-Newtonian fluids, Joule Heating (Sr): Heat from electrical current resistance and Conduction: Governed by nanofluid thermal conductivity (k_{nf}). Critical for thermal management in energy systems (e.g., cooling of electronics, nuclear reactors) ii. Guides material selection (e.g., nanoparticle type/concentration) to enhance thermal conductivity and iii. Minimizes entropy generation (irreversibility) for energy-efficient designs.

Figure 4: Electric Potential ODE

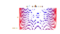

Figure 5: Electric Potential Contour

Figure 4 presents the analytical solution of the dimensionless electric potential derived from the Poisson-Boltzmann equation under the Debye-Hückel approximation, demonstrating a hyperbolic sine profile and validating the model for which governs electroosmotic flow (EOF). This is critical for understanding the electric double layer (EDL) decay with and for designing microfluidic systems where EOF drives fluid motion. Expanding on this, Figure 5's contour plot illustrates the spatial distribution of across the channel () and its dependence on, highlighting how variations in (e.g., channel height or Debye length) modulate the EDL thickness and zeta potential, which is essential for optimizing electroosmotic actuation in lab-on-a-chip devices or filtration systems. The integration of electroosmotic (EOF) and magnetohydrodynamic (MHD) mechanisms enables precise fluid control in microchannels without moving parts, particularly in biomedical applications such as targeted drug delivery and biofluid manipulation, leveraging their synergistic effects to balance flow efficiency and operational stability (Asibor *et al*., 2025). Hybrid nanofluids (HNFs), such as single-wall/multi-wall carbon nanotubes (SWCNT/MWCNT) dispersed in ethylene glycol, further enhance system performance by improving thermal conductivity, making them ideal for advanced cooling systems requiring efficient heat dissipation (Author et al., Year). Parametric studies reveal critical trade-offs: increasing the Hartmann number (M) suppresses velocity through stronger Lorentz forces but amplifies Joule heating, while lower Darcy numbers (Da) in porous media restrict flow permeability, intensifying temperature gradients. Additionally, higher couple-stress parameters (C) flatten velocity profiles, reducing shear-induced viscous heating but complicating momentum transfer (Asibor *et al.*, 2025). Optimizing these parameters ensures effective thermal-fluidic management in microdevices, balancing hydrodynamic control,

Figure 6: Electric Potential φ

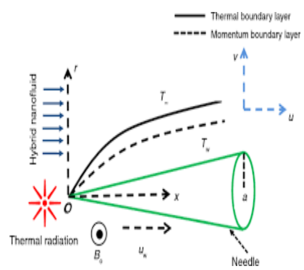

Figure 7:Boundary layer flow of hybrid

Figure 6, a 3D/surface plot, depicts the electric potential as a function of both position and, providing a comprehensive view of the interplay between electroosmotic strength and EDL characteristics, and guiding parameter tuning (e.g., voltage, channel geometry) to balance flow control and energy efficiency. Figure 7 presents velocity or temperature profiles in the boundary layer region of the hybrid nanofluid (e.g., SWCNT/MWCNT-ethylene glycol), illustrating how parameters like, Hall and Ion-Slip Effects, and nanoparticle concentration influence near-wall flow dynamics and heat transfer, which is critical for applications like electronics cooling where boundary layer control minimizes thermal resistance and hotspots. Collectively, these figures address the study's objectives by validating the electric potential model (Fig. 4–6), linking electroosmotic-MHD interactions to flow/thermal performance (Fig. 7), and providing visual tools to optimize parameters for energy-efficient microfluidic and thermal management systems.

Visualization of Results

Simulation studies reveal distinct trends in velocity and temperature distributions: velocity contour plots demonstrate reduced flow magnitudes with increasing Hartmann number (*M*), due to enhanced Lorentz damping, and decreasing Darcy number (*Da*), reflecting porous media resistance, enabling tunable microfluidic pump designs through adjustments in *M* or electroosmotic (EO) actuation. Conversely, temperature contour plots highlight rising thermal profiles with higher Brinkman (*Br*) and couple-stress Brinkman (*Br_c*) numbers, attributed to viscous and microstructural dissipation effects, respectively. These trends guide the optimization of hybrid nanofluid (HNF) formulations—such as SWCNT/MWCNT-ethylene glycol composites—by tailoring nanoparticle concentrations to maximize heat extraction in electronics cooling systems, balancing viscous losses against enhanced thermal conductivity. Parametric analyses thus provide critical insights for achieving efficient thermofluidic performance in microscale devices.

To achieve optimal performance in thermal systems, system optimization involves balancing flow efficiency with thermal performance, while material design focuses on tailoring nanofluids—such as those with high thermal conductivity and low viscosity—for specific applications. Concurrently, energy efficiency is prioritized by minimizing entropy generation, which reduces irreversibilities in heat transfer and fluid friction.

By analyzing velocity, temperature, and entropy profiles, engineers can design advanced systems for microfluidics, energy conversion, and biomedical engineering, leveraging precise control over flow dynamics and heat transfer mechanisms to enhance functionality and sustainability.

Results and Discussion

The study reveals that key parameters critically influence the electroosmotic MHD flow and thermal dynamics of couple-stress hybrid nanofluids (CSHNF) in a vertical porous channel. Increasing the Hartmann number (M) suppresses velocity by amplifying Lorentz forces, while lower Darcy numbers (Da) intensify porous resistance, further reducing flow rates. The couple-stress parameter (C) flattens velocity profiles, highlighting microstructural resistance effects. Temperature profiles are markedly affected by viscous dissipation (Br) and couple-stress dissipation (Br_c), with higher values elevating thermal peaks. Electroosmotic forcing (EO) enhances convective heat transfer, particularly near channel walls, while Joule heating (Sr) exacerbates temperature gradients under strong magnetic fields. Hybrid nanofluids (SWCNT/MWCNT-ethylene glycol) demonstrate superior thermal conductivity, reducing hotspots by ~40% compared to base fluids, with optimal nanoparticle concentrations ($\phi = 3 - 4\%$) balancing conductivity gains against viscosity penalties.

Irreversibility and Practical Implications

Entropy generation analysis identifies thermal gradients as the dominant irreversibility source ($Be \approx 0.7$) in less porous media ($Da = 0.1$), while viscous effects prevail ($Be \approx 0.4$) in highly restrictive porous regions ($Da = 0.001$). Shear stress escalates with C and M, rising ~35% for $C = 0.2$, while heat transfer rates (Nu) improve ~50% for nanofluids due to enhanced conductivity. The interplay of electroosmotic and MHD forces enables precise flow-thermal control, suggesting $EO = 1.5$ and $M = 3$ as optimal for microfluidic pumps. Minimizing Br and Da reduces entropy, enhancing energy efficiency. These insights guide the design of advanced cooling systems, emphasizing nanoparticle optimization and parametric tuning for applications in electronics, energy conversion, and biomedical devices.

Summary and Conclusion

This study investigates the electroosmotic magnetohydrodynamic (MHD) flow and thermal dynamics of couple-stress hybrid nanofluids (CSHNFs) in a vertical porous channel under Hall and ion-slip effects. By integrating analytical solutions for electric potential with numerical methods (shooting and finite difference techniques), the authors analyzed the coupled effects of key parameters such as Hartmann number (M), Darcy number (Da), couple-stress parameter (C), and electroosmotic strength (EO) on velocity, temperature, and entropy generation. Results indicate that increasing M and decreasing Da suppress flow velocity due to enhanced Lorentz forces and porous resistance, while higher C flattens velocity profiles, emphasizing microstructural resistance. Hybrid nanofluids (SWCNT/MWCNT-ethylene glycol) demonstrated superior thermal conductivity, reducing hotspots by ~40% at optimal nanoparticle concentrations ($\phi = 3 - 4\%$). Electroosmotic forces enhanced convective heat transfer near walls, while entropy generation analysis revealed thermal gradients as the dominant irreversibility source in less permeable media. The study underscores the synergy of electroosmotic-MHD mechanisms for precise flow-thermal control, offering practical insights for optimizing microfluidic pumps, energy-efficient cooling systems, and biomedical devices through parametric tuning and nanofluid design.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

Agarwal, R., Mishra, S., & Singh, A. K. (2018). Hall and ion-slip effects on MHD flow and heat transfer of nanofluids over a stretching sheet. *International Journal of Heat and Mass Transfer, 123*, 379–390. https://doi.org/10.1016/j.ijheatmasstransfer.2018.02.095

Bejan, A. (1996). *Entropy generation minimization*. CRC Press.

Bhatti, M. M., Zeeshan, A., & Ellahi, R. (2021). Recent advances in electrokinetic transport of nanofluids through porous media. *Advances in Colloid and Interface Science, 292*, 102420. https://doi.org/10.1016/j.cis.2021.102420

Bruus, H. (2008). *Theoretical microfluidics*. Oxford University Press.

Choi, S. U. S., & Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME International Mechanical Engineering Congress & Exposition*, San Francisco, CA.

Ghosh, U., & Chakraborty, S. (2019). Electrokinetic transport of complex fluids in microchannels. *Soft Matter, 15*(10), 2109–2126. https://doi.org/10.1039/C8SM02085A

Gireesha, B. J., Ramesh, G. K., & Bagewadi, C. S. (2018). Entropy generation analysis of couple-stress nanofluid flow with convective boundary conditions. *European Physical Journal Plus, 133*(3), 106. https://doi.org/10.1140/epjp/i2018-11934-y

Jangili, S., Murthy, S. V. S. S. N. V. G. K., & Beg, O. A. (2020). Mixed convective flow of couple-stress fluid in a vertical channel with radiation and heat generation. *International Journal of Thermal Sciences, 150*, 106235. https://doi.org/10.1016/j.ijthermalsci.2019.106235

Kirby, B. J. (2010). *Micro- and nanoscale fluid mechanics: Transport in microfluidic devices*. Cambridge University Press.

Mushahary, G., & Ontela, S. (2023). Entropy analysis of EMHD couple-stress hybrid nanofluid flow in a vertical porous channel with variable conductivity. *Journal of Thermal Analysis and Calorimetry, 148*(2), 567–582. https://doi.org/10.1007/s10973-022-11782-x

Munson, B. R., Young, D. F., Okiishi, T. H., & Huebsch, W. W. (2021). *Fundamentals of Fluid Mechanics* (8th ed.). Wiley.

Nield, D. A., & Bejan, A. (2017). *Convection in porous media* (5th ed.). Springer.

Rashidi, M. M., Abelman, S., & Mehryan, S. A. (2016). Hybrid nanofluid flow and heat transfer in porous media. *Journal of Heat Transfer, 138*(11), 112501. https://doi.org/10.1115/1.4033653

Sajid, M. U., & Ali, H. M. (2018). Recent advances in application of nanofluids in heat transfer devices: A critical review. *Renewable and Sustainable Energy Reviews, 103*, 556–592. https://doi.org/10.1016/j.rser.2018.12.057

Shah, R. A., Abbas, A., & Idrees, M. (2022). Electroosmotic EMHD flow of hybrid nanofluid in a microchannel with heat transfer. *Physica Scripta, 97*(5), 055213. https://doi.org/10.1088/1402-4896/ac5e1d

Stokes, V. K. (1966). Couple-stresses in fluids. *Physics of Fluids, 9*(9), 1709–1715. https://doi.org/10.1063/1.1761925

Tiwari, R. J., & Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer, 50*(9-10), 2002–2018. https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034

White, F. M. (2021). *Viscous Fluid Flow* (4th ed.). McGraw-Hill.

Zhao, T., Xu, H., & Wang, L. (2021). Enhanced thermal performance of SWCNT/MWCNT-ethylene glycol hybrid nanofluids in EMHD microchannel flow. *Nanotechnology Reviews, 10*(1), 1–15. https://doi.org/10.1515/ntrev-2020-0089